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MISCELLANEOUS PAPER NO. 6-746

# A RESONANT VIBRATION TECHNIQUE FOR LABORATORY DETERMINATION OF SHEAR WAVE VELOCITY

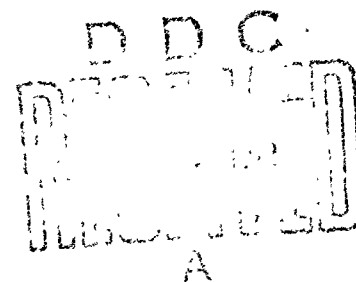
by

E. E. McCoy, Jr.



October 1965

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U. S. Army Engineer Waterways Experiment Station  
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### Foreword

This paper was prepared for submittal to the American Society for Testing and Materials for consideration for publication in "Materials Research and Standards."

The work was performed at the U. S. Army Engineer Waterways Experiment Station (WES) Concrete Division under the general supervision of Mr. Thomas B. Kennedy, Chief of the Division.

Director of the WES during the investigation and the preparation of this report was Col. John R. Oswalt, Jr., CE. Technical Director was Mr. J. B. Tiffany.

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A RESONANT VIBRATION TECHNIQUE FOR LABORATORY  
DETERMINATION OF SHEAR WAVE VELOCITY

by

E. E. McCoy, Jr.\*

Introduction

1. Interest in laboratory determinations of shear wave velocity has increased in recent years. A simple procedure permits the determination of the shear wave velocity of materials represented by laboratory specimens from data obtained with resonant-frequency testing equipment which is available in many laboratories. The method is applicable for either cylinder or prism specimens. The only resonant test data needed is the frequency of the first mode of the torsional vibration. Weights, densities, compressional wave velocities, and elastic moduli, each a separate source of error, are not used in this determination.

2. This paper discusses the test apparatus, test procedure, and the results obtained from typical shear-velocity tests employing the resonant vibration technique.

Test Apparatus

3. The test apparatus consists of the customary resonant-frequency test equipment; i.e. an audio oscillator, an output amplifier, a driver with short stylus, a pickup, a pickup amplifier, a meter, an oscilloscope, and an electronic counter. The counter is not required if the oscillator is calibrated to the percentage accuracy desired for the determination of shear velocity. Descriptions of such apparatus have been published.<sup>1,2,3\*\*</sup>

4. A phonographic pickup, with counterbalance arm, is most satisfactory for the meticulous examination of specimens. The directional

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\*\* Raised numerals refer to similarly numbered items in Literature Cited at end of text.

characteristic of a pickup of this type can be used to advantage, as explained subsequently, to determine the proper mode of vibration. A small, simple oscilloscope is adequate; however, it should have a switch to permit the use of either a constant-sweep circuit (60 cps) or the variable frequency of the audio output when a Lissajous display is desired.<sup>1</sup> The amplified pickup voltage provides the vertical component of the oscilloscope pattern.

#### Test Procedure

5. The procedure for determining torsional frequency is given in several papers as well as in standard test methods.<sup>1,2,4</sup> A sketch of the test setup is shown in fig. 1. The frequencies of the fundamental modes of flexural, torsional, and longitudinal vibration are usually obtained during resonance testing, but only the torsional frequency is required for the determination of shear wave velocity. Much skill is required to determine which resonant frequency is being indicated during a test. Special techniques to determine these frequencies have been published;<sup>1,4</sup> those for torsional frequency are reviewed below.

6. Since only the torsional frequency is to be obtained in the determination of shear velocity, great care should be exercised to ensure that torsional resonance is in fact indicated and that it is in the proper mode. This is done by successively probing with the pickup along the full length of the specimen. There will be only one node, i.e. at the center of the specimen; consequently, the amplitude indicated by the meter or oscilloscope will be maximum at the ends and minimum at the center. With a Lissajous display, the continuous phase change from one end to the other can be observed. Since the same is true for longitudinal resonance, a further precaution is necessary. By probing with the pickup across the end of the specimen, a complete distinction between the two types of resonance can be made.<sup>1</sup> No difference in response will be noted across the end for longitudinal resonance, but a change in amplitude and phase, similar to that observed along the length of the specimen, will be observed for torsional resonance. With a directionally sensitive pickup, a further distinction can be made. The vibratory motion of the specimen for longitudinal resonance is greater in the direction of the axis, whereas

the motion for torsional resonance is greater normal to the axis. This difference can readily be detected by changing the orientation of the pickup.

### Theory

7. For the first mode of torsional resonant vibration, a standing half wave (fig. 1) is present in the specimen.<sup>5</sup> The shear velocity in the specimen then is  $2\pi fl$ ,  $f$  being the resonant torsional frequency and  $l$  the length of the specimen. However, this velocity is specifically the shear velocity in the particular specimen tested and is not necessarily the same for large masses of the material represented by the test specimen. No further calculation is necessary for a circular cylinder test specimen, but for a rectangular prism a shape factor must be used, as shown subsequently.

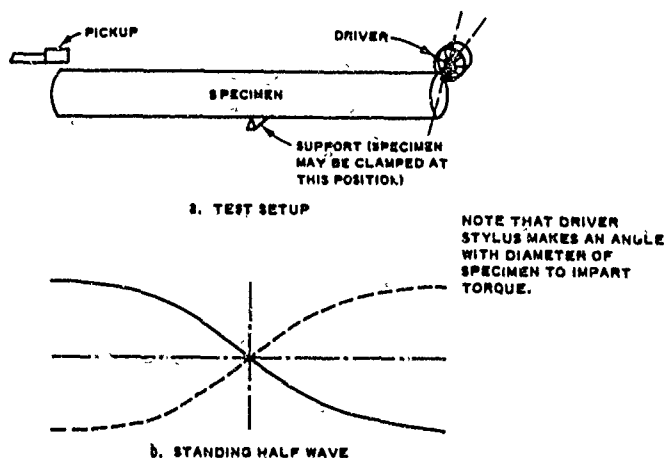


Fig. 1. Test setup and wave form

### Derivation of Formulas

8. The formulas used for the determination of shear velocity from resonant, torsional frequencies were obtained by equating the two following well-known expressions for shear modulus. The first (from references 6 and 7) is

$$G = \rho v_t^2$$

and the second (from reference 4) is

$$G = \frac{W}{B} f^2$$

where

$G$  = shear modulus

$\rho$  = density

$V_t$  = shear velocity

$B$  = constant given by Pickett<sup>4</sup>

$W$  = weight of specimen

$f$  = frequency of fundamental mode of resonant, torsional vibration

The formulas are:

a. For a cylinder,

$$V_t = 2f\ell$$

b. For a prism of square cross section,

$$V_t = 1.0878 (2f\ell) = 2.1756f\ell$$

c. For a prism of generally rectangular cross section,

$$V_t = 2f\ell \sqrt{\frac{a/b + b/a}{4a/b - 2.52(a/b)^2 + 0.21(a/b)^6}}$$

where  $a$  and  $b$  are cross-sectional dimensions with  $a$  being less than  $b$ .

#### Longitudinal Resonance Analogy

9. It is interesting to note that an analogous derivation from the longitudinal resonant frequency gives,<sup>6</sup>

$$V_c \cong 2f\ell \sqrt{\frac{1 - \sigma}{(1 + \sigma)(1 - 2\sigma)}}$$

where  $V_c$  is the compressional wave velocity for the material, and  $2f\ell$  is the specific compressional wave velocity in the specimen with  $\ell$  being the length of the specimen and  $f$  its longitudinal resonant frequency.



The formula should be used only for specimens having lengths several times greater than their cross-sectional dimensions.<sup>6</sup> In this simple formula, the velocity  $V_c$ , though independent of shape, is highly dependent on Poisson's ratio  $\sigma$ . The relation provides a method for the determination of Poisson's ratio when an independent determination of  $V_c$  is available.

Results of Typical Shear-Velocity Tests  
Using Resonant Vibration Method

10. Typical shear velocities obtained in resonant vibration tests on four specimens are as follows:

<u>Specimen</u>	<u>Specimen Dimensions, ft</u>	<u>Torsional Frequency cps</u>	<u>Shape Factor</u>	<u>Shear Velocity <math>V_t</math> fps</u>
Concrete prism	0.293 × 0.375 × 1.333	2750	1.124	8,240
Grout cylinder	0.250 diam × 1.250	1890	1.000	4,725
Limestone rock core	0.208 diam × 1.120	4930	1.000	11,045
Aluminum prism	0.250 × 0.333 × 1.417	3280	1.134	10,540

11. The aluminum tested<sup>8</sup> was 24ST, and had a Poisson's ratio of 0.33 and a compressional wave velocity of 20,800 fps (calculated by the formula given in paragraph 9). Agreement between the three quantities  $V_t$ ,  $V_c$ , and  $\sigma$  for the aluminum specimen was excellent when checked by a graphical method.<sup>9</sup> Textbook values<sup>10</sup> for 17ST aluminum of  $V_t = 10,165$  fps,  $V_c = 20,500$  fps, and  $\sigma = 0.355$  were somewhat less compatible and are considered to be possibly representative values rather than the results of tests on a single bar of aluminum.

Conclusion

12. In summary, with a knowledge of only the frequency of the fundamental mode of resonant, torsional vibration of a specimen and the length of the specimen, the specific shear velocity of the specimen was found to

be equal to  $2f\ell$ , which multiplied by a shape factor is the shear-wave velocity of the material represented by the specimen. The accuracy of results is dependent upon, and approximately equivalent to, the accuracy of the two basic relations from which the formulas for shear velocity were derived.

13. An analogous derivation for the longitudinal resonant frequency provides an approximate method for the determination of Poisson's ratio.

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